

Strange Stars in $f(T)$ Gravity With MIT Bag Model

G. Abbas^{a *} Shahid Qaisar^{a †} and Abdul Jawad^{b ‡}

^aDepartment of Mathematics, COMSATS
Institute of Information Technology, Sahiwal-57000, Pakistan.

^bDepartment of Mathematics, COMSATS
Institute of Information Technology, Lahore, Pakistan.

Abstract

This paper deals with existence of strange stars in $f(T)$ modified gravity. For this purpose, we have taken the diagonal tetrad field of static spacetime with charged anisotropic fluid and MIT bag model, which provide the linear relation between radial pressure and density of the matter. Further, the analysis of the resulting equations have been done by assuming the parametric form of the metric functions in term of the radial profiles with some unknown constant (introduced by Krori and Barua). By the matching of two metrics, unknown constant of the metric functions appear in terms of mass, radius and charge of the stars, the observed values of these quantities have been used for the detail analysis of the the derived model. We have discuss the regularity, anisotropy, energy conditions, stability and surface redshift of the model.

Keywords: $f(T)$ Gravity; Exact solutions; Strange stars; MIT Bag Model.

PACS: 04.40.Dg; 04.20.Jb; 04.20.Dw

*abbasg91@yahoo.com

†shahidqaisar90@ciitsahiwal.edu.pk

‡sweetfriend181@gmail.com

1 Introduction

Recently, the modified theories of gravitation have attracted the attention of a huge community of theoretical physicists due to their implication as an alternative framework for the successful proposal of dark energy. This modification is carried out in several ways by modifying the Lagrangian of General Relativity (GR), for examples Ricci scalar R in the Einstein-Hilbert action must be replaced by $f(R)$ (see Durrer and Maartens 2010; DE Felice and Tsujikawa 20210). In GR and modified theories of gravity, the gravitational effects could be described in term of contents of matter and curvature of spacetime. However, one can formulate an analogous theory of GR, which can be demonstrated in term of torsion instead of curvature. Originally, Einstein formulated this theory to unify the basic forces due to Coulomb field and gravitational field Einstein (1928). This problem of unification remains unsolved in the new theory, but after many years the theory becomes much popular as an alternative theory of GR, this theory is known as teleparallel equivalent of GR (TEGR) Möller (1961). The basic concept to formulate this theory is to take a such manifold that preserve torsion along with curvature. The complete Riemann curvature tensor can be considered as zero, hence in this way gravitational field can be described by torsion. The appropriate technique is to utilize the tetrad fields e^i_μ Weitzenböck (1923).

During the current years, the researchers have carried out the study of TEGR with some modification to explore some theoretical as well experimental problems of modern cosmology (Ferraro and Fiorini 2007, 2008). The comparison of $f(R)$ gravity with $f(T)$ gravity implies that later one is more convenient as in $f(T)$ gravity, equations of motions are of second order while for metric $f(R)$ gravity, these equations are fourth order. Recently, a lot of work in $f(T)$ gravity is devoted to investigate the observed data of the theoretical cosmology or to determine the constraint on the parameter of theory, so that the theoretical results may fit with the data obtained by some standard source like WMAP etc. Further, many researchers have investigated that a particular class of $f(T)$ gravity models can explain some basic problems of cosmology like inflation and accelerated expansion (Bengochea and Ferraro 2009 Wu and Yu 2010).

In addition to study the cosmology, $f(T)$ theory has been used to examined the effects of $f(T)$ models on the existence of Black hole and compact objects in 3-dimensions. On the basis of this fact, the BTZ black hole model has been derived in $f(T)$ theory of gravity. (Zheng and Huang 2011). Also, it

has been proved that the first of black hole thermodynamics in $f(T)$ gravity becomes invalid, due to the break down of Lorentz invariance (Bengochea 2011). Further, some static vacuum solutions with charged source have been investigated in $f(T)$ theory, such solutions leads to charged black hole solutions in $f(T)$ gravity (Li et al.2011). A large class of static perfect fluid solutions exhibit the existence of relativistic stars in $f(T)$ gravity (Dent et al.2011, Rong-Xin et al.2011; Wang et al. 2011, Boehmer et al. 2011, Daouda 2011).

It has been the subject of great interest to study the models of anisotropic stars during the last decades in GR as well in modified theories of gravity (Herrera and Santos 1997, Chan et al.1993, Di Prisco 1997, Herrera and Barreto (2013), Abbas (2014a), (2014b), (2014c), (2014d), Abbas and Sabiullah (2014)). Mak and Harko (2004) present a class of regular and analytic solution of field equation with anisotropic source. Also, Chaisi and Maharaj (2005) have discussed the nature of anisotropic matter source, after establishing an algorithm. By utilizing the Chaplygin gas equation of state (EOS), Rahaman et al. (2012) generalized the Krori-Barua (KB) (1975) solutions to the matter source with electric field. It has become a scientific tool to study the models the compact stars with Krori-Barua metric (Kalam et al. 2012, Kalam et al. 2013). Hossein et al. (2012) discussed the modelling of the compact star model by taking cosmological constant as radial coordinate dependent. Bhar et al. (2015) and Maurya et al.(2015) discuss the possibility for the existence of higher dimensional and charged compact stars, respectively.

In present year, Abbas and his collaborators (Abbas et al.2014, Abbas et al.2015a, 2015b, 2015c, 2015d) have studied the models of anisotropic compact stars in GR, $f(R)$, $f(G)$ and $f(T)$ theories in diagonal tetrad case using Krori and Barua (KB) metric approach. Das et al.(2015) have examined the existence of compact stars in $f(T)$ gravity using the conformal motion technique and have discussed properties of the resulting models. The main objective of the present paper is to study the models of the anisotropic compact stars in the context of $f(T)$ gravity using diagonal tetrad in the presence of electric field and MIT Bag Model. We have examined the anisotropic behavior, regularity at the center as well as stability of these models. Finally, the surface redshift has been calculated. All these properties of the models have been discussed by using the observational data of the compact stars. The plan of the paper is the following: In the next section, we present the review of $f(T)$ gravity and equations of motion with charged anisotropic.

Section 3 deals with charged anisotropic source and the equation of motion in $f(T)$ gravity with diagonal tetrad. Section 4 investigates with the physical analysis of the proposed models. In the last section, we conclude the results of the paper.

2 $f(T)$ Theory of Gravity

Currently, Teleparallel theory of gravity is an equivalent theory of gravitation to the GR (Zhang et al.2011, Li et al. 2011). In this section, we shall introduce the basic concepts of the $f(T)$ gravity theory. To this end, we define notation of the Latin indices for the tetrad field and Greek indices for spacetime coordinates. The line element of the manifold is given by

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu. \quad (1)$$

This spacetime can be transformed to a Minkowskian form by the matrix transformation as follows:

$$dS^2 = g_{\mu\nu}dx^\mu dx^\nu = \eta_{ij}\theta^i\theta^j, \quad (2)$$

$$dx^\mu = e_i^\mu\theta^i, \theta^i = e_\mu^i dx^\mu, \quad (3)$$

where $\eta_{ij} = \text{diag}[1, -1, -1, -1]$ and $e_i^\mu e_i^\nu = \delta_\nu^\mu$ or $e_i^\mu e_j^\nu = \delta_i^j$. The root of the metric determinant is given by $\sqrt{-g} = \det[e_\mu^i] = e$.

We consider the manifold for which the Riemann tensor is zero and non-zero torsion terms exist, the Weitzenbock's connection components can be defined as follows:

$$\Gamma_{\mu\nu}^\alpha = e_i^\alpha \partial_\nu e_\mu^i. \quad (4)$$

We define the torsion and the contorsion tensor as follows:

$$T_{\mu\nu}^\alpha = \Gamma_{\nu\mu}^\alpha - \Gamma_{\mu\nu}^\alpha = e_i^\alpha (\partial_\mu e_\nu^i - \partial_\nu e_\mu^i), \quad (5)$$

$$K_\alpha^{\mu\nu} = -\frac{1}{2}(T_\alpha^{\mu\nu} - T_\alpha^{\nu\mu} - T_\alpha^{\mu\nu}), \quad (6)$$

and the components tensor $S_\alpha^{\mu\nu}$ are defined as

$$S_\alpha^{\mu\nu} = \frac{1}{2}(K_\alpha^{\mu\nu} + \delta_\alpha^\mu T_\beta^{\beta\nu} - \delta_\alpha^\nu T_\beta^{\beta\mu}), \quad (7)$$

one can write the torsion scalar as

$$T = T_{\mu\nu}^\alpha S_\alpha^{\mu\nu}. \quad (8)$$

Now, similarly to the $f(R)$ gravity, one defines the action of $f(T)$ theory as follows:

$$S[e_\mu^i, \Phi_A] = \int d^4x e \left[\frac{1}{16\pi} f(T) + \mathcal{L}_{Matter}(\Phi_A) \right], \quad (9)$$

where we used $G = c = 1$ and Φ_A is matter fields. Now the variation of above action yields the following form of equations of motion (Wu and Yu 2010 and Zheng and Huang 2011)

$$S_\mu^{\nu\rho} \partial_\rho T f_{TT} + [e^{-1} e_\mu^i \partial_\rho (e e_i^\alpha S_\alpha^{\nu\rho}) + T_{\lambda\mu}^\alpha S_\alpha^{\nu\lambda}] f_T + \frac{1}{4} \delta_\mu^\nu f = 4\pi (\mathcal{T}_\mu^\nu + E_\mu^\nu), \quad (10)$$

where \mathcal{T}_μ^ν and E_μ^ν are energy momentum tensors of ordinary matter and electromagnetic field.

The ordinary matter is an anisotropic fluid for which the energy-momentum tensor is given by

$$\mathcal{T}_\mu^\nu = (\rho + p_t) u_\mu u^\nu - p_t \delta_\mu^\nu + (p_r - p_t) v_\mu v^\nu, \quad (11)$$

where u^μ is the four-velocity, v^μ radial four vector, ρ the energy density, p_r is the radial pressure and p_t is transverse pressure. Further, the energy momentum tensor for electromagnetic field is given by

$$E_\mu^\nu = \frac{1}{4\pi} (g^{\delta\omega} F_{\mu\delta} F^\nu{}_\omega - \frac{1}{4} g_\mu^\nu F_{\delta\omega} F^{\delta\omega}), \quad (12)$$

where $F_{\mu\nu}$ is the Maxwell field tensor defined as

$$F_{\mu\nu} = \phi_{\nu,\mu} - \phi_{\mu,\nu} \quad (13)$$

and ϕ_μ is the four potential.

3 Equation of Motion in $f(T)$ Gravity

We assume that the interior spacetime of a strange star is describe by the KB (1975) metric

$$ds^2 = -e^{a(r)}dt^2 + e^{b(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (14)$$

In order to re-write above line element, into the invariant form under the Lorentz transformations, we define the tetrad matrix as

$$[e_\mu^i] = \text{diag}[e^{\frac{a(r)}{2}}, e^{\frac{b(r)}{2}}, r, r\sin(\theta)]. \quad (15)$$

Further, one can obtain $e = \det[e_\mu^i] = e^{\frac{(a+b)}{2}}r^2\sin(\theta)$. For the charged fluid source with density $\rho(r)$ radial pressure $p_r(r)$, tangential pressure $p_t(r)$. The Einstein-Maxwell (EM) equations take the form (with geometrized units $G = c = 1$).

$$T(r) = \frac{2e^{-b}}{r}(a' + \frac{1}{r}), \quad (16)$$

$$T'(r) = \frac{2e^{-b}}{r}(a'' + \frac{1}{r^2} - T(b' + \frac{1}{r})), \quad (17)$$

where the prime ' denotes the derivative with respect to the radial coordinate r . One can now re-write the equations of motion for anisotropic fluid as

$$4\pi\rho + E^2 = \frac{f}{4} - \left(T - \frac{1}{r^2} - \frac{e^{-b}}{r}(a' + b')\right) \frac{f_T}{2}, \quad (18)$$

$$4\pi p_r - E^2 = \left(T - \frac{1}{r^2}\right) \frac{f_T}{2} - \frac{f}{4}, \quad (19)$$

$$4\pi p_t + E^2 = \left[\frac{T}{2} + e^{-b}\left(\frac{a''}{2} + \left(\frac{a'}{4} + \frac{1}{2r}\right)(a' - b')\right)\right] \frac{f_T}{2} - \frac{f}{4}, \quad (20)$$

$$\frac{\cot\theta}{2r^2}T'f_{TT} = 0, \quad (21)$$

$$E(r) = \frac{1}{r} \int_0^r 4\pi r'^2 \sigma e^{\frac{\lambda}{2}} dr' = \frac{q(r)}{r^2} \quad (22)$$

where $q(r)$ is the total charge within a sphere of radius r . According to the MIT bag model, we take the simple form of the strange matter EoS (Rahman et al.(2012))

$$p_r = \frac{1}{3}(\rho - 4B_g). \quad (23)$$

From Eq.(21), we get the following linear form of $f(T)$ function

$$f(T) = \beta T + \beta_1, \quad (24)$$

where β and β_1 are integration constants, for simplicity we assume $\beta_1 = 0$. We parameterize the metric as follows (Krori and Barua 1975)

$$b(r) = Ar^2, \quad a(r) = Br^2 + C, \quad (25)$$

where A , B and C are some arbitrary constants which will be determined later using some physical conditions. With the choice of the EoS, we have a system of five independent equations with five unknown parameters namely, ρ , p_r , p_t , $E(r)$ and $\sigma(r)$. From Eqs.(18)-(25), we obtain

$$\rho = \frac{3(A+B)}{16\pi} e^{-Ar^2} \beta + B_g, \quad (26)$$

$$p_r = \frac{(A+B)}{16\pi} e^{-Ar^2} \beta - B_g, \quad (27)$$

$$p_t = \frac{1}{8\pi} \left[\left(\frac{7}{2}B - \frac{3}{2}A + B^2r^2 - AB r^2 + \frac{1}{r^2} \right) e^{-Ar^2} \beta - \frac{1}{r^2} \right] + B_g, \quad (28)$$

$$E^2 = \frac{1}{2} \left(A - 3B - \frac{2}{r^2} \right) e^{-Ar^2} \beta + \frac{\beta}{r^2} - 8\pi B_g, \quad (29)$$

The charge density is obtained as

$$\sigma = \frac{e^{-\frac{Ar^2}{2}}}{2\pi r} \psi + \frac{A e^{-\frac{3Ar^2}{2}}}{8\pi r \psi} \left[2 - (A - 3B) r^2 \right] + \frac{e^{-\frac{Ar^2}{2}}}{4\pi r^3 \psi} \left(e^{-Ar^2} - 1 \right), \quad (30)$$

where

$$\psi = \sqrt{\left[\frac{1}{2} \left(A - 3B - \frac{2}{r^2} \right) e^{-Ar^2} \beta + \frac{\beta}{r^2} - 8\pi B_g \right]} \quad (31)$$

The amount of net charge inside a sphere of radius r is given by

$$q = r^2 \sqrt{\left[\frac{1}{2} \left(A - 3B - \frac{2}{r^2} \right) e^{-Ar^2} \beta + \frac{\beta}{r^2} - 8\pi B_g \right]} \quad (32)$$

4 Physical Analysis of Model

4.1 Central Regularity of Model Parameters

It well know fact that the KB metric preserve no singularity in its metric functions even at the center at $r = 0$. Therefore, it becomes necessary to impose the constraints on the constants appearing in the metric functions, so that physical parameters of the model remain well behaved at all the inner points strange stars. For the regularity at the center ($r = 0$), we obtain the central density in the form

$$\rho_0 = \rho(r = 0) = \frac{3(A + B)}{16\pi}\beta + B_g. \quad (33)$$

Further, the regularity of electric field requires that it must vanish at the center i.e.,

$$E^2(r = 0) = \frac{3}{2}(A - B) - 8\pi B_g = 0, \quad (34)$$

which implies that

$$B_g = \frac{3(A - B)}{16\pi}\beta \quad (35)$$

The pressures and density should be decreasing function of r . In our model, radial variation of p_r is obtained as

$$\frac{dp_r}{dr} = -\frac{(A + B)}{8\pi}rAe^{-Ar^2}\beta < 0, \quad (36)$$

At $r = 0$, $\frac{dp_r}{dr} = 0$ and $\frac{d^2p_r}{dr^2} < 0$. This implies that p_r will decrease radially outward. The radial variation of matter energy density is obtained as

$$\frac{d\rho}{dr} = -\frac{3(A + B)}{8\pi}rAe^{-Ar^2}\beta \quad (37)$$

which also shows that at $r = 0$, $\frac{d\rho}{dr} = 0$ and $\frac{d^2\rho}{dr^2} = -\frac{3}{8\pi}\beta(A + B) < 0$. Hence density is decreasing function of r , provided that $\beta > 0$, as we have chosen it in our discussion. Hence, it has been shown graphically that pressures and energy density decreases as shown in figures **1-3**. The anisotropic stress is obtained as

$$\begin{aligned} \Delta &= p_t - p_r = 2B_g - \frac{\beta}{8\pi r^2} \\ &+ \frac{1}{8\pi}(3B - 2A + B^2r^2 - ABr^2 + \frac{1}{r^2})e^{-Ar^2}\beta. \end{aligned} \quad (38)$$

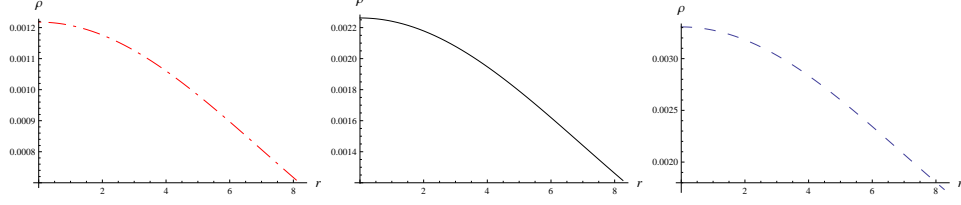


Figure 1: This figure represents the variation of ρ versus $r(km)$ for a strange star of radius 8.26 km with $\beta = 1$, $\beta = 2$ and $\beta = 3$. The corresponding values of A , B and B_g have been used from table 2. All the graphs have been plotted for table 2, it will not be mentioned again.

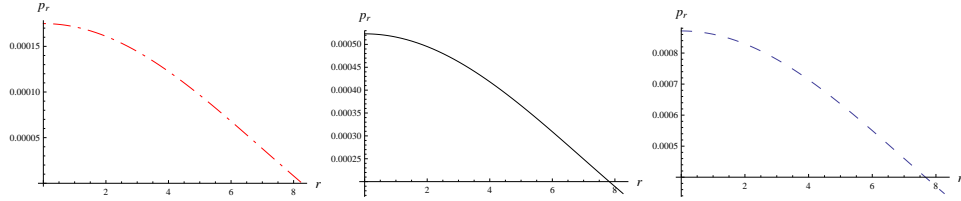


Figure 2: This figure represents the variation of p_r versus $r(km)$ for a strange star of radius 8.26 km with $\beta = 1$, $\beta = 2$ and $\beta = 3$.

The anisotropic force of the gravitating system will be outwardly directed when $p_t > p_r$ i.e., $\Delta > 0$, while inwardly directed when $p_t < p_r$ i.e., $\Delta < 0$. It is clear from the figure 4 that there exist repulsive gravitational force for $1 \leq \beta \leq 2$ as $(\Delta > 0)$, while for $\beta \geq 3$ force becomes attractive. In the case of repulsive force more massive distribution of matter would exist, which is the result of disturbance of equilibrium. Figure 5 implies that E^2 is increasing function of radial coordinate for all the values of β .

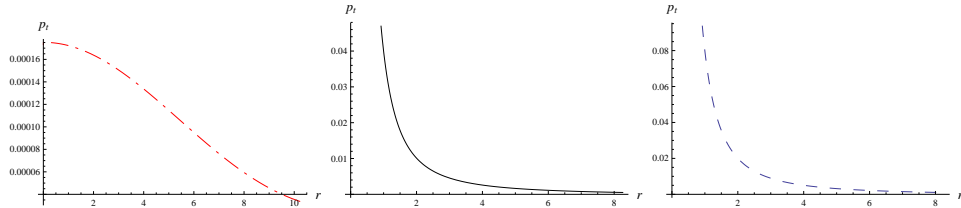


Figure 3: This figure represents the variation of p_t versus $r(km)$ for a strange star of radius 8.26 km with $\beta = 1$, $\beta = 2$ and $\beta = 3$.

4.2 Matching Conditions

The exterior region of the star is taken as Reissner-Nordstrom metric given by

$$ds^2 = - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} \right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (39)$$

where Q is the total charge within boundary $r = R$. Continuity of the metric coefficients g_{tt} , g_{rr} and $\frac{\partial g_{tt}}{\partial r}$ across the boundary surface $r = R$ between the interior and the exterior regions of the star yields the following results:

$$1 - \frac{2M}{R} + \frac{Q^2}{R^2} = e^{BR^2+C} \quad (40)$$

$$1 - \frac{2M}{R} + \frac{Q^2}{R^2} = e^{-AR^2} \quad (41)$$

$$\frac{M}{R^2} - \frac{Q^2}{R^3} = BRe^{BR^2+C} \quad (42)$$

By solving the above three equations, we get

$$A = -\frac{1}{R^2} \ln \left[1 - \frac{2M}{R} + \frac{Q^2}{R^2} \right] \quad (43)$$

$$B = \frac{1}{R^2} \left[\frac{M}{R^2} - \frac{Q^2}{R^2} \right] \left[1 - \frac{2M}{R} + \frac{Q^2}{R^2} \right]^{-1} \quad (44)$$

$$C = \ln \left[1 - \frac{2M}{R} + \frac{Q^2}{R^2} \right] - \frac{\frac{M}{R} - \frac{Q^2}{R^2}}{\left[1 - \frac{2M}{R} + \frac{Q^2}{R^2} \right]} \quad (45)$$

For the given values of the parameters M, R and Q the values of A and B are given in table 2.

4.3 Energy conditions

The anisotropic charged fluid considered as strange matter will satisfy the null energy condition (NEC), weak energy condition (WEC) and strong energy condition (SEC) if the following inequalities hold simultaneously at all points for the given radius of the star:

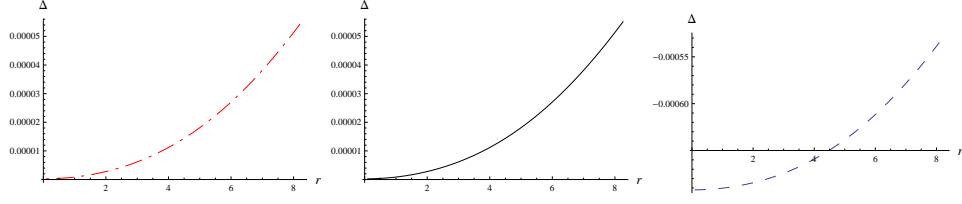


Figure 4: This figure represents the variation of Δ versus $r(km)$ for a strange star of radius 8.26 km with $\beta = 1$, $\beta = 2$ and $\beta = 3$.

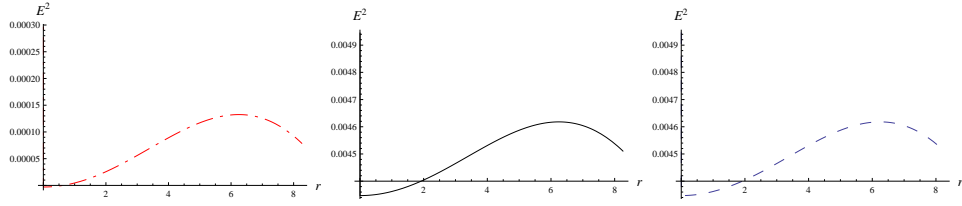


Figure 5: This figure represents the variation of E^2 versus $r(km)$ for a strange star of radius 8.26 km with $\beta = 1$, $\beta = 2$ and $\beta = 3$.

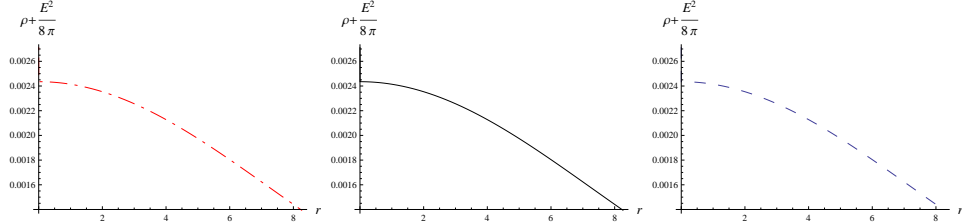


Figure 6: This figure represents the variation of $\rho + \frac{E^2}{8\pi}$ versus $r(km)$ for a strange star of radius 8.26 km with $\beta = 1$, $\beta = 2$ and $\beta = 3$.

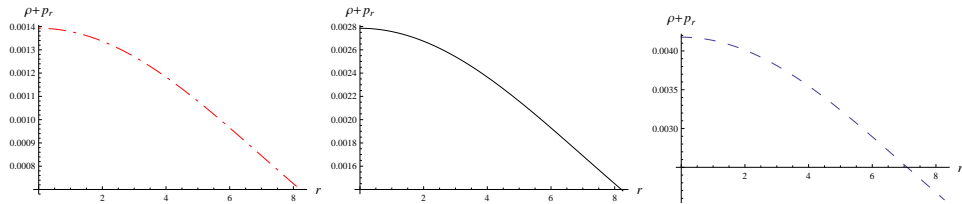


Figure 7: This figure represents the variation of $\rho + p_r$ versus $r(km)$ for a strange star of radius 8.26 km with $\beta = 1$, $\beta = 2$ and $\beta = 3$.

Case	M	$R(km)$	$\frac{M}{R}$	$\frac{Q^2}{R^2}$
1	$1.4M_{\odot}$	8.26	0.25	0.004
2	$1.4M_{\odot}$	6.88	0.30	0.027
3	$1.4M_{\odot}$	5.90	0.35	0.061
4	$1.4M_{\odot}$	5.16	0.40	0.105

Table 1: Values of M, R and Q defined in Rahman et al.(2012)

Case	$A (km^{-2})$	$B (km^{-2})$	$\mathbf{B}_g (km^{-2})$
1	0.0102	0.0073	0.0001732
2	0.01798	0.01351	0.000267
3	0.0292	0.0231	0.0003643
4	0.044	0.037	0.000418

Table 2: Values of the model parameters A, B and B_g for given Masses and Radii of Stars

$$\rho + \frac{E^2}{8\pi} \geq 0, \quad (46)$$

$$\rho + p_r \geq 0, \quad (47)$$

$$\rho + p_t + \frac{E^2}{4\pi} \geq 0, \quad (48)$$

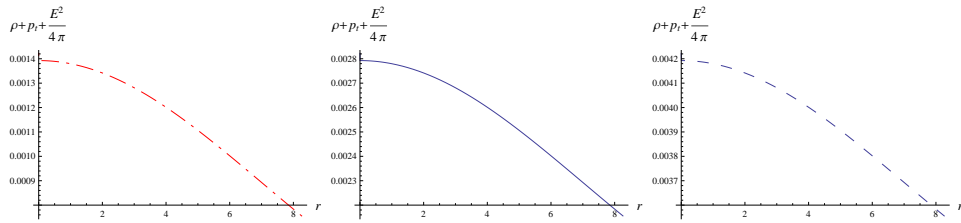


Figure 8: This figure represents the variation of $\rho + p_r + \frac{E^2}{4\pi}$ versus $r(km)$ for a strange star of radius 8.26 km with $\beta = 1$, $\beta = 2$ and $\beta = 3$.

Case	$\rho(r=0) (gm\ cm^{-3})$	$\rho(r=R) (gm\ cm^{-3})$	$\mathbf{p}_r (dyne\ cm^{-2})$
1	1.643×10^{15}	9.362×10^{14}	2.123×10^{35}
2	2.895×10^{15}	1.443×10^{15}	4.361×10^{35}
3	4.703×10^{15}	2.015×10^{15}	8.204×10^{35}
4	7.087×10^{15}	2.58×10^{15}	14.48×10^{35}

Table 3: Values of density and pressure at center and surface of star for $\beta = 1$

Case	$\rho(r=0) (gm\ cm^{-3})$	$\rho(r=R) (gm\ cm^{-3})$	$\mathbf{p}_r (dyne\ cm^{-2})$
1	1.643×10^{15}	9.362×10^{14}	2.123×10^{35}
2	2.895×10^{15}	1.443×10^{15}	4.361×10^{35}
3	4.703×10^{15}	2.015×10^{15}	8.204×10^{35}
4	7.087×10^{15}	2.58×10^{15}	14.48×10^{35}

Table 4: Values of density and pressure at center and surface of star for $\beta = 2$

$$\rho + p_r + 2p_t + \frac{E^2}{4\pi} \geq 0, \quad (49)$$

Employing these energy conditions at the centre ($r = 0$), we get the following bounds on the constants A and B :

$$(i) NEC : \rho + \frac{E^2}{8\pi} \geq 0 \Rightarrow A \geq 0.$$

$$(ii) WEC : \rho + p_r \geq 0 \Rightarrow A + B \geq 0, \rho + p_t + \frac{E^2}{4\pi} \geq 0 \Rightarrow A + B \geq 0$$

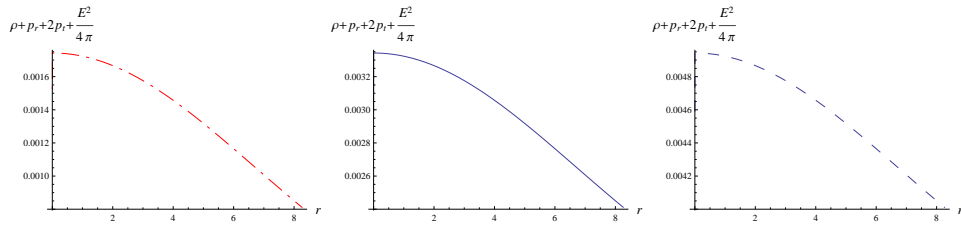


Figure 9: This figure represents the variation of $\rho + p_r + 2p_t + \frac{E^2}{4\pi}$ versus $r(km)$ for a strange star of radius 8.26 km with $\beta = 1$, $\beta = 2$ and $\beta = 3$.

Case	$\rho(r=0) (gm\ cm^{-3})$	$\rho(r=R) (gm\ cm^{-3})$	$\mathbf{p}_r (dyne\ cm^{-2})$
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Table 5: Values of density and pressure at center and surface of star for $\beta = 3$

$$(iii) SEC : \rho + p_r + 2p_t + \frac{E^2}{4\pi} \geq 0 \Rightarrow B \geq 0.$$

Since $A > 0$, so condition (i) is satisfied. The weak and strong energy conditions (ii) and (iii) will be satisfied if $B \geq 0$. With the set of values given in table 1 and 2, we have shown in Figures 6-9 that the energy conditions are valid through out the interior of the given star.

4.4 Stability

In order to discuss the stability of our constructed models, we follow the approach based on Herrera (1992) cracking (or overturning) concept. According to this technique the squares of the radial and tangential sound speeds must lie in the interval $[0,1]$. Further, cracking concepts predicts that the region for which the radial speed of sound is greater than that of transverse speed, then such region is potentially stable region. It is clear that, for no cracking (stability), the difference of two sound speeds, i.e., $v_{st}^2 - v_{sr}^2$ should attain the same sign everywhere inside the anisotropic matter distribution. In our model, we have

$$v_{sr}^2 = \frac{dp_r}{d\rho} = \frac{1}{3}, \quad (50)$$

$$v_{st}^2 = \frac{dp_t}{d\rho} = \frac{\alpha + \gamma}{-3(A + B)rA\beta e^{-Ar^2}}, \quad (51)$$

where,

$$\alpha = e^{-Ar^2} \beta \left(2B^2r - 2ABr - \frac{2}{r^3} \right) + \frac{1}{r^2} \quad (52)$$

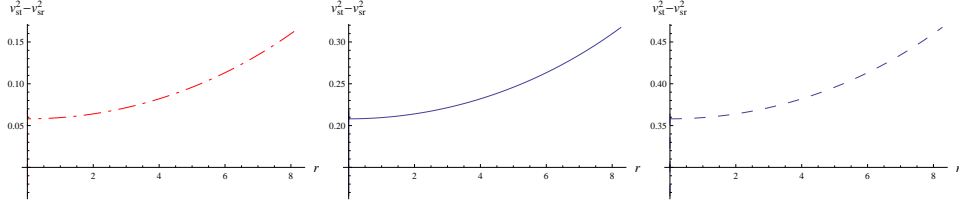


Figure 10: This figure represents the variation of $v_{st}^2 - v_{sr}^2$ versus $r(km)$ for a strange star of radius 8.26 km with $\beta = 1$, $\beta = 2$ and $\beta = 3$.

$$\gamma = -2A r e^{-Ar^2} \beta \left(\frac{7}{2}B - \frac{3}{2}A + B^2 r^2 - A B r^2 + \frac{1}{r^2} \right) \quad (53)$$

For causality condition to be satisfied, we must have

$$0 < \frac{\alpha + \gamma}{-3(A + B)r A \beta e^{-Ar^2}} < 1 \quad (54)$$

For the assumed set of values, figure **10** shows that the above condition is satisfied. Hence, our proposed model preserve stability in $f(T)$ gravity .

4.5 Surface Redshift

We define the compactification factor as

$$u = \frac{M_{eff}}{R} = \frac{1}{2}\beta \left(1 - e^{-Ar^2} \right)$$

The surface red-shift (Z_s) corresponding to the above compactness (u) is obtained as

$$Z_s = \left(1 - \beta \left(1 - e^{-Ar^2} \right) \right)^{-\frac{1}{2}} - 1$$

The maximum surface red-shift, in this set up, for a strange star of mass $1.4M_\odot$ and radius $8.26km$ turns out to be $Z_s = 1.4$ for $\beta = 1$, while $Z_s < 1.4$ for $\beta > 1$ as shown in Figure **11**.

5 Discussion

During the last few years, theoretical physicists have shown their attraction to study gravitational field as the effect of torsion instead of curvature of

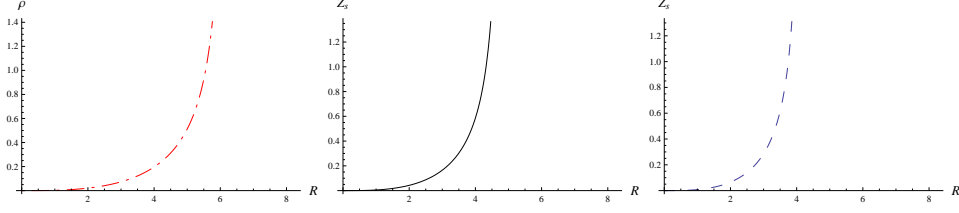


Figure 11: This figure represents the variation of redshift Z_s versus $r(km)$ for a strange star of radius 8.26 km with $\beta = 1$, $\beta = 2$ and $\beta = 3$.

the underlying geometry. The torsion theory was originally derived in parallel lines to general theory of relativity which describes gravity in term of curvature. Hence, this idea was promoted as teleparallel equivalence of GR. The black hole (BH) and neutron stars are the highly dense astronomical objects which store information about the entropy on the BH horizon. In the modified TEGR, $f(T)$ gravity posses many significant feature as compared to GR. Recent observations from solar system orbital motions in order to constrain $f(T)$ gravity have been made and interesting results have been found. The general constraints for the existence/non-existence of compact stars have been studied in $f(T)$ gravity (Boehmer et al. 2011). It is interesting to drive the models of the compact stars in $f(T)$ gravity using the diagonal tetrad field with charged anisotropic fluid and MIT Bag model.

This paper deals with construction of analytic models of compact stars in $f(T)$ gravity with charged anisotropic matter and MIT Bag EoS. The inner region of the stars have been taken as static charged anisotropic spherical source. By using the diagonal tetrad field, the equations of motion have been formulated to complete the discussion. One of the field equations implies that unknown function $f(T)$ appears as a linear function of T i.e., $f(T) = \beta T + \beta_1$, where β and β_1 are the constant of integration. Using this form of $f(T)$ with KB metric functions, we have determined explicitly the matter components and electric field intensity. The anisotropy, regularity and energy conditions have been discussed in detail. The observed values of masses, radii and charge of compact stars have been used to calculate the unknown constants of KB metric. The first and second derivatives of density and pressures, implies the maximality of these quantities at the center, and these have decreasing radial profile. The values of density and pressure at $r = 0$ and $r = R$ are given in tables **3-5**.

We have found that (see Figure 4) that there exist repulsive gravitational

force for $1 \leq \beta \leq 2$ as ($\Delta > 0$), while for $\beta \geq 3$ force becomes attractive. The first case regarding to repulsive gravitational force leads to the construction of more massive object, while second case gives the formation of smaller objects. The Herrera's cracking concept states that the region for which the radial speed of sound is greater than that of transverse speed, then such region is potentially stable region. It is clear that for stability, the difference of two sound speeds, i.e., $v_{st}^2 - v_{sr}^2$ should attain the same sign everywhere inside the anisotropic matter distribution. In our discussion, the compact stars remains stable in $f(T)$ gravity even in the presence of electromagnetic field. The maximum surface redshift, for a strange star of mass $1.4M_{\odot}$ and radius $8.26km$ in $f(T)$ gravity turns out to be $Z_s = 1.4$ for $\beta = 1$, while $Z_s < 1.4$ for $\beta > 1$ as shown in Figure 11.

6 Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this work.

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